Exercise Sheet 5

Exercise 17 Maximum Likelihood Estimation

Determine a maximum likelihood estimator for the parameter θ of a uniform distribution on the interval $[0, \theta]$! Reminder: the random variables underlying the sample vector have the probability density function

$$f_X(x; \theta) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{\theta}, & \text{if } 0 \leq x \leq \theta, \\ 0, & \text{if } x > \theta. \end{cases}$$

Check whether the resulting estimator is consistent and unbiased!
(Hint: When maximing the likelihood function bear in mind that certain frame conditions have to be satisfied. Compare your result to the estimators discussed in the lecture.)

Exercise 18 Maximum Likelihood Estimation

From an urn with N balls of which M balls are red and $N - M$ are black five balls are drawn with replacement. Assume the result was three red balls and two black balls. Derive a maximum likelihood estimator for the number (or fraction) of red balls M in dependence of N. You may ignore that the number should be integer! **Hint:** Start by setting up the likelihood function $\mathcal{L}(M)$ and look for a maximum of this function.

Exercise 19 Confidence Intervals

In the year 1972 45195 of the 87827 live births in Lower Saxony were boys. From this data, determine a point estimator for the unknown probability p that a newly born child is a boy, as well as confidence intervals for the confidence levels

a) $\alpha = 0.01$ (99% confidence interval) and
b) $\alpha = 0.001$ (99.9% confidence interval).

(Hint: The needed quantiles of the normal distribution may be computed with the C program `ndqtl.c`, which is available on the lecture's WWW page. Quantile: argument value corresponding to a given function value of a distribution function; analogous to the quantiles of a sample.)

Exercise 20 Confidence Intervals

Starting from the point estimator for the parameter θ of an exponential distribution that was already considered in Exercise 13, that is, $W_2 = n \min_{i=1}^{n} X_i$, determine a confidence interval for this parameter! Reminder: In Exercise 13 we derived

$$f_{W_2}(w;\theta) = \frac{1}{\theta} e^{-\frac{w}{\theta}}$$

as the probability density function of the estimator W_2.