Artificial Neural Networks and Deep Learning Summer 2017
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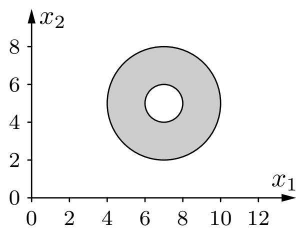
Exercise Sheet 4

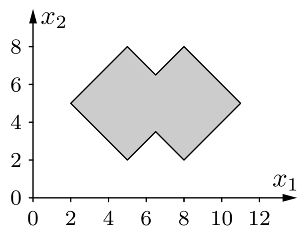
Exercise 22 Radial Basis Function Networks: Binary Functions

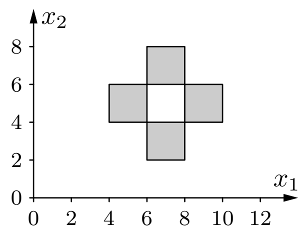
Determine the parameters (weights $\vec{w}_u$ and radii σ_u) of radial basis function networks with the activation function

$$f_{\text{act}}^{(u)}(\text{net}_u, \sigma_u) = \begin{cases} 1, & \text{if } \text{net}_u \leq \sigma_u, \\ 0, & \text{otherwise,} \end{cases}$$

for the neurons in the hidden layer, that produce the value 1 for points inside the gray areas of the diagrams shown below and the value 0 outside! It does not matter whether the networks produce a value of 0 or a value of 1 for points on the boundaries of the gray areas. However, you should make sure that for eavery point in the x_1-x_2 plane *either* a value of 0 *or* a value of 1 is computed.

a)


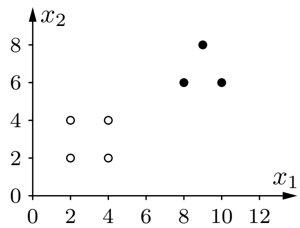
b)


c)


Exercise 23 Radial Basis Function Networks: Two Class Problems

Determining the weights of the connections from the input neurons to the neurons of the hidden layer — that is, determining the centers of the radial basis functions — and determining the radii are among the main problems of training radial basis function networks. For classification problems, statistical estimation procedures are sometimes employed to find suitable (initial values for the) centers and radii, at least if it can be expected that a single radial basis function per class suffices. In order to do so, one interprets the radial basis function as a scaled probability density and determines the expected value and the standard deviation of the distribution e.g. with maximum likelihood estimation.

As an example we consider a radial basis function network with two inputs, two hidden neurons and two output neurons, which is supposed to classify the data set shown on the right. The hidden neurons have the Euclidean distance as their network input function and the Gaussian function $f_{\text{act}}(\text{net}, \sigma) = e^{-\frac{\text{net}^2}{2\sigma^2}}$ as their activation function. Determine suitable centers $\vec{w}$ and radii σ for the two classes with the help of maximum likelihood estimation! What has to be taken into account when computing the radii?



Exercise 24 Radial Basis Function Networks: Function Approximation

a) Construct a radial basis function network with about 10 neurons that approximates the function $y = x^2$ in the interval $[0.5, 4.5]$ by a step function!

b) How can the approximation be improved? (State at least two possibilities.)

Exercise 25 Radial Basis Function Networks: Initialization

Determine the parameters (weights $\vec{w}_u$ and bias values θ_u) of a simple radial basis function network that computes the implication $x_1 \rightarrow x_2$! All basis functions should have the radius $\frac{3}{2}$. The hidden neurons should have the maximum distance as their network input function and a triangular function

$$f_{\text{act}}(\text{net}_u, \sigma_u) = \begin{cases} 0, & \text{if } \text{net}_u > \sigma_u, \\ 1 - \frac{\text{net}_u}{\sigma_u}, & \text{otherwise.} \end{cases}$$

as their activation function.

Exercise 26 Radial Basis Function Networks: Initialization

Using the method of the (Moore–Penrose) pseudo-inverse, determine the parameters (weights $\vec{w}_u$ and bias values θ_u) of radial basis function networks that compute the conjunction $x_1 \wedge x_2$! Employ

a) two radial basis functions mit centers $(0, 0)$ and $(1, 1)$,
b) one radial basis function with center $(1, 1)$.

All basis functionen should have the radius $\frac{1}{2}$. The hidden neurons should have the Euclidean distance as their network input function and a Gaussian function

$$f_{\text{act}}(\text{net}_u, \sigma_u) = e^{-\frac{\text{net}_u^2}{2\sigma_u^2}}$$

as their activation function. Compute the actual output of the two networks and compare it to the desired outputs! Why do we obtain a perfect solution of the learning problem in part a)?

Exercise 27 Radial Basis Function Networks: Initialization

Using the method of the (Moore–Penrose) pseudo-inverse, determine the parameters (weights $\vec{w}_u$ and bias values θ_u) of radial basis function networks that compute the Exclusive Or $x_1 \dot{\vee} x_2$ (or $x_1 \oplus x_2$)! Employ

a) two radial basis functions mit centers $(0, 0)$ and $(1, 1)$,
b) one radial basis function with center $(1, 1)$.

All basis functions should have the radius $\frac{5}{4}$. The hidden neurons should have the city block distance (also known as Manhattan distance) as their network input function and a triangular function as their activation function. Compute the actual output of the two networks and compare it to the desired outputs!