Artificial Neural Networks and Deep Learning Summer 2017
Christian Borgelt from 2017.05.02

Organisational Rules for Exam Admission

The lecture is accompanied by exercise sheets. At the beginning of each exercise lesson, the exercises are *voted for*. By voting for an exercise, one expresses one's willingness to present something about it. (Suggestions for a solution will be discussed, they need not be correct right away.) To the exam will be admitted who

1. voted for *at least* half of the exercises *and*
2. presented something for *at least* two exercises.

Exercise Sheet 1

Exercise 1 Distance of a Point from a Plane

Find a formula to compute the distance of a given point $\vec{y} = (y_1, y_2, y_3)^\top$ to a plane $(\vec{x} - \vec{p})^\top \vec{n} = 0$. Use this formula to compute the distance for $\vec{p} = (-1, -1, -1)^\top$, $\vec{n} = (4, -2, 3)^\top$ and $\vec{y} = (3, 14, -6)^\top$.

Exercise 2 Threshold Logic Units

Determine the parameters of *single* threshold logic units in such a way that these units compute the following Boolean functions:

a) $y = x_1 \vee x_2$ b) $y = \neg x_1 \wedge x_2$

(Hint: A threshold logic unit computes on which side of a (hyper-)plane a given input vector lies.)

Exercise 3 Threshold Logic Units

Try to find parameters of a threshold logic unit in such a way that it computes the Exclusive Or (written $x_1 \dot{\vee} x_2$ or $x_1 \oplus x_2$)! What problem does one run into? How can one solve this problem? (Hint: Recall the solution of the biimplication problem that was studied in the lecture.)

Exercise 4 Threshold Logic Units

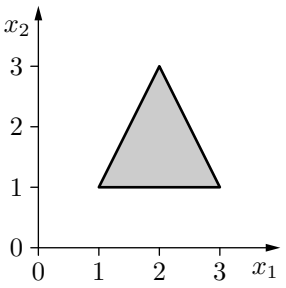
Determine the parameters of *single* threshold logic units in such a way that they compute the following Boolean functions:

a) $y = x_1 \wedge \neg x_2 \wedge x_3$ (or short: $x_1 \overline{x_2} x_3$)
b) $y = (x_1 \wedge \neg x_2) \vee (x_1 \wedge x_3)$ (or short: $x_1 \overline{x_2} \vee x_1 x_3$)
c) $y = (x_1 \wedge x_2) \vee (\neg x_2 \wedge x_3)$ (or short: $x_1 x_2 \vee \overline{x_2} x_3$)
d) $y = (x_1 \wedge x_2) \vee \neg x_3$ (or short: $x_1 x_2 \vee \overline{x_3}$)

(Hint: A threshold logic unit computes on which side of a (hyper-)plane a given input vector lies.)

Exercise 5 Networks of Threshold Logic Units

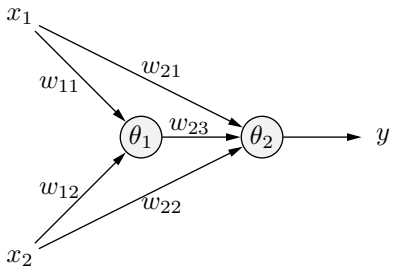
Construct a neural network of threshold logic units that produces the output 1 for points (x_1, x_2) inside of the triangle shown in the sketch and the output 0 for points outside!



(Hint: Recall the neural network for the biimplication problem that was studied in the lecture and interpret the computations of the threshold logic units in the first layer as a coordinate transformation.)

Exercise 6 Networks of Threshold Logic Units

Determine the parameters w_{ji} and θ_j of the neural network of threshold logic units that is shown in the sketch on the right in such a way that it computes the Exclusive Or of the Boolean variables x_1 and x_2 (that is, $y = x_1 \dot{\vee} x_2$ or $y = x_1 \oplus x_2$)!



(Hint: Start from a geometric interpretation of the computation in the input space of the neuron on the right and consider how you may use the output of the left neuron to arrange the points (x_1, x_2), for which 1 or 0 should be produced, in such a way that they become separable by a plane.)