Fuzzy Systems
Prof. Dr. Rudolf Kruse, Christian Moewes

Assignment Sheet 8

Assignment 29 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ be two sets and $\mu : X \to [0, 1]$ and $\nu : Y \to [0, 1]$ two fuzzy sets on X and Y, respectively, which are defined as follows:

$$\begin{array}{llll} \mu(x_1) = 0.1, & \mu(x_2) = 0.7, & \mu(x_3) = 1.0, & \\ \nu(y_1) = 0.4, & \nu(y_2) = 1.0, & \nu(y_3) = 0.8, & \nu(y_4) = 0.3. \end{array}$$

a) How can you find out whether the relational equation $\mu \circ \varrho = \nu$ has a solution, *i.e.* whether there is a fuzzy relation ϱ that satisfies this equation?

b) If the relational equation $\mu \circ \varrho = \nu$ has a solution, determine a solution. Are there other solutions than the one you found?

Assignment 30 Fuzzy Relational Equations

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be two sets. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown in the two tables below.

	x_1	x_2	x_3
μ_1	1.0	0.6	0.2
μ_2	0.0	0.8	1.0
μ_3	0.9	0.1	0.0

	y_1	y_2
ν_1	1.0	0.4
ν_2	0.6	1.0
ν_3	0.9	0.5

a) Show that the system consisting of the two relational equations $\mu_1 \circ \varrho = \nu_1$ and $\mu_2 \circ \varrho = \nu_2$ has a solution. Find the greatest solution of this system.

b) Is the fuzzy relation that can be computed as the union (maximum) of the two Cartesian products $\mu_1 \otimes \nu_1$ and $\mu_2 \otimes \nu_2$ also a solution of the system of relational equations considered in a)?

c) Show that the system consisting of the three relational equations $\mu_i \circ \varrho = \nu_i$, $i = 1, 2, 3$, does not have any solution.

Assignment 31 Fuzzy Control based on Relational Equations

Let $X = \{1, 2, 3\}$ and $Y = \{10, 20, 30\}$ be two sets, μ_1, μ_2 fuzzy sets on X, and ν_1, ν_2 fuzzy sets on Y, which are defined as shown in the two tables below.

	1	2	3
μ_1	0.0	0.5	1.0
μ_2	1.0	0.4	0.0

	10	20	30
ν_1	0.0	0.6	1.0
ν_2	1.0	0.3	0.0

Consider a fuzzy controller with the following rule base:

if x **is** μ_1 **then** y **is** ν_1**,**
if x **is** μ_2 **then** y **is** ν_2**.**

Use the Gödel relation to determine the fuzzy output of this controller for the fuzzy input $(1 : 0.1, 2 : 1, 3 : 0)$.

Assignment 32 Programming

Implement the concept of a controller based on a system of relational equations.

a) Enable the user to enter finite crisp sets X and Y with an arbitrary number of elements.
b) Let the user enter corresponding fuzzy sets $\mu_1, \ldots, \mu_r$ on X and $\nu_1, \ldots, \nu_r$ on Y.
c) Write a function that computes the greatest solution for each $\mu_i \circ \varrho = \nu_i$.
d) Write a method that outputs the greatest solution for all $\mu_i \circ \varrho = \nu_i$.
e) Write a main method that combines a) to d) so that, *e.g.* Assignment 30 can be solved.