Assignment Sheet 5

Assignment 17 Fuzzy Implication

In the lecture we considered 9 axioms that a fuzzy implication I should satisfy, namely

1. $a \leq b$ implies $I(a,x) \geq I(b,x)$ *(monotonicity in 1st argument)*
2. $a \leq b$ implies $I(x,a) \leq I(x,b)$ *(monotonicity in 2nd argument)*
3. $I(0,a) = 1$ *(dominance of falsity)*
4. $I(1,b) = b$ *(neutrality of truth)*
5. $I(a,a) = 1$ *(identity)*
6. $I(a, I(b,c)) = I(b, I(a,c))$ *(exchange property)*
7. $I(a,b) = 1$ if and only if $a \leq b$ *(boundary condition)*
8. $I(a,b) = I(\sim b, \sim a)$ for fuzzy complement $\sim$ *(contraposition)*
9. I is a continuous function *(continuity)*

We also studied different fuzzy implications, but not all of them satisfy all of these conditions. In this assignment we check some of the assertions made in the lecture.

a) Show explicitly that $I_{Ł}(a,b) = \min(1,\ 1 - a + b)$ satisfies all Axioms 1–9.

b) Show that $I_Z(a,b) = \max[1-a, \min(a,b)]$ does not satisfy Axioms 5–8.

c) Show that $I_{\min}(a,b) = \begin{cases} 1, & \text{if } a \leq b \\ b & \text{otherwise.} \end{cases}$ does not satisfy Axioms 8 and 9.

Assignment 18 Generation of Fuzzy Implications

Which class of fuzzy implications do you obtain by applying the following theorem to linear functions?

Theorem: *A function $I : [0,1]^2 \to [0,1]$ satisfies Axioms 1–9 (see Assignment 17) of fuzzy implications for a particular fuzzy complement $\sim$ if and only if there exists a strict increasing continuous function $f : [0,1] \to [0,\infty)$ s.t. $f(0) = 0$,*

$$I(a,b) = f^{(-1)}(f(1) - f(a) + f(b))$$

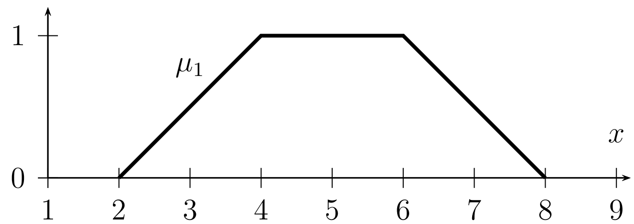
for all $a, b \in [0,1]$, and

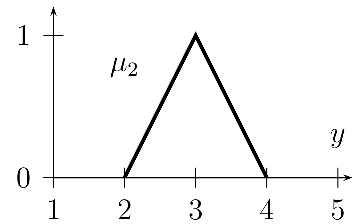
$$\sim a = f^{-1}(f(1) - f(a))$$

for all $a \in [0,1]$.

Assignment 19 The Extension Principle

Consider the following two fuzzy sets:





Use the extension principle to apply the following two functions to these fuzzy sets:

a) $z = \frac{1}{x}$

b) $z = x - 2y$

Draw a sketch of the resulting fuzzy sets on the domain of z.

Assignment 20 Set Representation and Extension Principle

Consider the following definition of triangular fuzzy numbers

$$\mu_{l,m,r} = \begin{cases} \frac{x-l}{m-l} & \text{if } l \le x \le m, \\ \frac{r-x}{r-m} & \text{if } m \le x \le r, \\ 0 & \text{otherwise} \end{cases}$$

whereas $l, m, r \in \mathbb{R}$ and $l < m < r$. Now, let $\mu_{1,2,3}$ be an interpretation of the vague concept "around 2".

a) Compute $\{5\} \oplus \mu_{1,2,3} \ominus \mu_{1,2,3}$ with the help of set representations.

b) Compute the extension $\hat{\phi}(\mu_{1,2,3})$ for $\phi(a) = 5 + a - a$.